Research Statement

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My research lies at the interface of analysis, geometry, topology, and mathematical physics. A central question guiding my work is: *how can techniques from global analysis and index theory be used to understand spectral problems and to construct and compare geometric and topological invariants?* A recurring theme in my work is to convert global or singular problems into local geometric analysis. Heat-kernel analysis connects local geometry with global spectral invariants, compactifications of configuration spaces provide a geometric framework for analyzing the singularities, and Witten deformation is a powerful analytic tool that simplifies spectral problems by concentrating the relevant information near critical sets. My research develops along three interconnected directions:

(a) Feynman graph integrals in holomorphic quantum field theory and their applications to higher-genus B-model invariants;

(b) analytic torsion and higher Cheeger-Müller theory, using Witten deformation to compare analytic and topological invariants;

(c) spectral theory of Schrödinger-type operators on noncompact manifolds, motivated in part by Landau-Ginzburg models in physics.

One long-term program brings these directions together toward a Calabi–Yau/Landau–Ginzburg correspondence on the B-model side for higher genus.

1 Overview of my results

Feynman graph integrals in holomorphic field theories. With Minghao Wang, we proved the convergence of Feynman graph integrals on real-analytic Kähler manifolds in the sense of Cauchy principal values [23]. On Calabi–Yau threefolds, this gives a direct and rigorous realization of the candidate higher-genus closed-string B-model invariants proposed by BCOV. This makes it possible to formulate higher-genus closed-string mirror symmetry conjecture. We subsequently proved that three rigorous renormalization procedures for holomorphic field theories give the same answer, an agreement that is not generally expected in QFT, and derived a gauge anomaly formula [24].

Analytic torsion and higher torsion. Using Witten deformation by specially chosen non-Morse functions, I developed a localization method adapted to decompositions of manifolds. This gives a new purely analytic proof of the gluing formula for analytic torsion and, for families, a simpler proof of the gluing formula for Bismut-Lott torsion forms [25, 26]. With Martin Puchol [19], we proved a comparison of analytic and topological higher torsions for unitary flat bundles using generalized Morse functions. This removes the fiberwise Morse-function existence assumption in [3] and trivial flat bundle assumption in [20], while retaining Thom–Smale transversality. The proof resolves the main analytic difficulty caused by birth–death singularities.

Spectral theory on noncompact manifolds. I proved that the weighted Laplacian on every complete gradient Ricci shrinker satisfies the classical Weyl law [27]. With Maxim Braverman and Xianzhe Dai, we obtained a Weyl-law criterion for Schrödinger operators on general complete manifolds [8], removing the bounded-geometry and doubling assumptions of our earlier result [12]. We also constructed examples in which the Weyl law fails when the criterion is violated [8]. My earlier work with Dai established Agmon estimates heat-kernel expansions and a local index theorem for Schrödinger-type operators on noncompact manifolds [10, 11].

2 Feynman graph integrals and holomorphic field theory

Background and question. In perturbative quantum field theory, Feynman graph integrals encode the contributions associated with interactions of particles. In mathematics, they also provide a striking bridge between physical theories and geometric or topological invariants: In topological quantum field theory (TQFT), such integrals converge in the Lebesgue sense and give rise to important topological invariants [1, 7, 17].

In general QFTs, however, graph integrals require renormalization, and different procedures may produce different values. Our work asks whether the geometry of holomorphic field theories on Kähler manifolds permits a canonical construction.

For a graph Γ with vertex set Γ_0 and edge set Γ_1, assign a point $x_v \in M$ to each vertex. Each edge $e = (v, w)$ contributes a propagator $P_e(x_v, x_w)$, constructed from a Green kernel and its derivatives. The graph integral has the schematic form

$$\int_{M^{\Gamma_0}} \prod_{e=(v,w)\in\Gamma_1} P_e(x_v, x_w),$$

where $M^{\Gamma_0} = \prod_{v\in\Gamma_0} M$. The propagators are singular when their endpoints coincide (i.e., $x_v \to x_w$). The main analytic difficulty is to control simultaneous and nested collisions of vertices.

Convergence. In joint work with Wang, we proved the convergence of these graph integrals on closed real-analytic Kähler manifolds [23]. Using heat-kernel techniques together with Getzler rescaling from local index theory, we show that the graph integrands extend canonically to the Fulton-MacPherson compactification of configuration spaces (the compactification records how points collide) as forms with controlled singularities and therefore admit well-defined Cauchy principal-value regularizations.

Applications and significance of our results. When specialized to Calabi–Yau threefolds, our construction gives a direct and mathematically rigorous definition of the candidate higher-genus ($g \geq 2$) B-model invariants through the Feynman graph integrals proposed by BCOV in [2]. In complex dimension one, work of Li [18] and Costello–Li [9] established a version of the all-genus mirror-symmetry correspondence for elliptic curves within the BCOV framework, giving strong evidence for the higher-genus mirror-symmetry picture envisioned by BCOV. Our result therefore makes it possible to formulate the corresponding higher-genus closed-string mirror-symmetry conjecture for higher dimensions in precise mathematical terms.

Agreement of three renormalization procedures. A further question is whether different renormalization procedures produce the same value. With Wang, we proved that heat-kernel renormalization, zeta regularization, and Cauchy principal-value regularization all give the same value for holomorphic field theories on closed real-analytic Kähler manifolds [24]. The difficulty is that these procedures organize the singularities in very different ways. We compare these procedures by constructing a wonderful compactification of the product of the configuration space with the complexified time parameters.

We also derived a gauge anomaly formula: the failure of the resulting graph currents to be $\bar{\partial}$-closed is expressed by boundary integrals over the compactified space of time parameters [24, §6]. This anomaly formula computes the obstruction to the quantum master equation in the BV formalism.

Future directions. We will next extend the construction to families and study anomaly formulas for Feynman graph integrals under variations of Kähler and complex structures. The goal is to understand the differential forms on moduli spaces of Calabi–Yau manifolds that arise from fiberwise Feynman graph integrals, in analogy with the analytic torsion forms discussed in §3. This would supply analytic tools for computing these amplitudes and for comparing them with Gromov–Witten invariants of the mirror Calabi–Yau manifold, as higher-genus mirror symmetry predicts. The main analytic problem is to obtain estimates on the propagators and their singularities that remain uniform in the family parameter.

A second, longer-term program concerns field theory on manifolds with boundary, where my Witten-deformation gluing methods may serve to separate bulk and boundary contributions. This would lay the analytic groundwork for gluing formulas in perturbative field theory, with the aim of reconstructing global quantum invariants from calculations on simpler pieces together with the data on their common boundary.

3 Higher Cheeger-Müller/Bismut-Zhang theory

Background. A classical problem in global analysis is to understand why spectral invariants of differential operators can coincide with invariants defined by topological or combinatorial methods. Ray-Singer analytic torsion is a central example: it is defined from regularized determinants of Hodge Laplacians, whereas Reidemeister torsion is constructed from a finite-dimensional cellular complex. The Cheeger-Müller theorem identifies analytic and Reidemeister torsion. Bismut–Zhang extended the comparison to a more general setting using Witten deformation, which deforms the de Rham differential by a Morse function so that small eigenforms localize at its critical points [6].

For families of manifolds, the corresponding problem is to compare differential-form-valued analytic torsion with higher topological torsion over the parameter space. Let $\pi : M \to S$ be a smooth fibration with fibers $Z_s = \pi^{-1}(s)$. Bismut and Lott generalized the numerical analytic torsion of a single manifold to an even differential form for a flat complex vector bundle $F \to M$

$$\mathcal{T}_{\mathrm{BL}}(M/S, F)$$

on the parameter space S [5]. Its degree-zero component recovers ordinary analytic torsion. On the topological side, Igusa and Klein constructed the topological analogue of higher analytic torsion, which can detect exotic smooth structures that are invisible to ordinary characteristic classes [13–15].

What was known and its limitations. A higher Cheeger-Müller/Bismut-Zhang theorem should identify the higher analytic torsion of Bismut-Lott with its topological counterpart. There are two main approaches to such a theorem, but both impose different and quite strong restrictions. Bismut-Goette's analytic proof [3] applies to general flat bundles, but requires a fiberwise Morse function satisfying fiberwise Thom-Smale transversality (a genericity condition making the Thom-Smale complex well defined). The axiomatic approach in Puchol-Zhang-Zhu [20] removes the existence assumption on fiberwise Morse functions, but is restricted to trivial flat bundles.

Difficulties and result obtained. With Martin Puchol, we prove the comparison for general unitary flat bundles without assuming the existence of a fiberwise Morse function [19], thereby substantially removing the restrictions inherent in both approaches above. The axiomatic results in [20] reduce the theorem to a relative comparison. On the analytic side, the relative higher torsion is

$$\mathcal{T}^{\mathrm{rel}}_{\mathrm{BL}}(M/S, F) := \mathcal{T}_{\mathrm{BL}}(M/S, F) - \mathcal{T}_{\mathrm{BL}}\big(M/S, \underline{\mathbb{C}}^{\mathrm{rk}(F)}\big),$$

where $\underline{\mathbb{C}}^{\mathrm{rk}(F)}$ is the trivial flat bundle of the same rank as F; the topological torsion is normalized in the same way.

The main difficulty is that this requires a new Witten-deformation analysis for fiberwise generalized Morse functions. Such functions always exist, but they may develop birth–death points, where the local form contains a cubic term such as x^3. As one approaches a fiber containing such a point, two critical points merge and the Hessian degenerates, so the estimates underlying the usual Morse analysis cease to be uniform. Establishing uniform estimates and controlling the resulting small eigenspaces therefore requires a substantial amount of delicate and technically demanding analysis.

We handle this by a localization argument adapted from the non-Morse Witten deformation I developed for torsion gluing formulas [25, 26], which isolates the contribution of the birth–death region. Since analytic torsion and torsion forms are defined through the heat kernels of the relevant operators, the analysis naturally separates into small- and large-time parts. In the small-time analysis, the isolated birth–death contribution cancels in the relative torsion against the corresponding contribution from

the trivial bundle.. For large time, we combine Bismut-Lebeau's techniques [4] with the noncompact Agmon estimates developed in my earlier joint work with Dai [10] to obtain the uniform estimates needed near the birth-death locus. Together with the axiomatic reduction above, this yields the higher Cheeger-Müller/Bismut-Zhang theorem for the higher torsions, while still *assuming fiberwise Thom-Smale transversality.*

Future direction: removing transversality by higher structures. My next goal is to remove global Thom-Smale transversality from this comparison theorem, paving the way toward a higher Cheeger-Müller/Bismut-Zhang theorem in full generality. Since the Thom-Smale condition can always be achieved locally on the base, I plan to choose vertical metrics satisfying it over a sufficiently fine open cover $S = \bigcup_i U_i$ and glue the resulting local Morse complexes by higher continuation data. On a double intersection $U_i \cap U_j$, two choices of metric are related by a continuation map. On higher intersections one must introduce higher homotopies.

This higher structure should define a Čech version of the topological torsion, while local Witten deformation should identify it with the Bismut-Lott analytic torsion form. The new issue is to control continuation maps and higher homotopies uniformly near birth–death loci, extending the estimates used in our previous comparison theorem.

4 Analysis of Schrödinger-type operators on noncompact manifolds

A basic problem in spectral geometry is to understand spectral invariants of Schrödinger-type operators on noncompact manifolds. This question also arises naturally in the Landau–Ginzburg B-model. An LG model consists of such a manifold X and a holomorphic function f, called the superpotential. The associated operator on differential forms has the form

$$\Delta_f = \Delta + |\nabla f|^2 + \text{a zero-order term involving } \operatorname{Hess}(f).$$

Regularized determinants of this operator are expected to govern the genus-one theory.

A major obstacle in this program is that many fundamental tools of spectral geometry rely heavily on compactness. In a series of joint works with X. Dai [10–12], we developed analytic foundations for Schrödinger-type operators on noncompact manifolds, which are needed to define higher-genus BCOV invariants in the LG B-model. We proved exponential decay of eigenfunctions near infinity (Agmon estimates), which leads to the well-definedness of Thom-Smale-Witten cohomology for Morse functions on noncompact manifolds [10]. We also derived heat-kernel asymptotic expansions for such operators [11], allowing us to extend the local index theorem to noncompact manifolds with potentials. Lastly, defining zeta-regularized determinants requires control of the eigenvalue growth, which is exactly what the Weyl law provides.

Weyl law for Schrödinger operators on noncompact manifolds. A basic question in spectral geometry is whether the classical Weyl law continues to hold for Schrödinger operators on noncompact manifolds, where both the geometry at infinity and the growth of the potential can affect the eigenvalue distribution.

Using the heat-kernel expansion developed in [11], we established in [12] a *non-semiclassical Weyl law* for Schrödinger operators on noncompact manifolds with bounded geometry and potentials satisfying a doubling condition. This gives a Weyl law in a broad noncompact geometric setting without prescribing a specific model for the geometry at infinity. Even in special cases, such as hyperbolic manifolds with rapidly growing potentials studied in [16], which fall within the scope of our result, the analysis is already delicate.

In subsequent work with Braverman and Dai, we removed both the bounded geometry and doubling assumptions [8]. Our intrinsic criterion balances local curvature and injectivity radius against the growth and oscillation of the potential. It allows geometric irregularity at infinity, provided that this balance is preserved. We also constructed examples in which the Weyl law fails when the balance breaks down.

More recently, I applied these ideas to show that the weighted Laplacian on every complete gradient Ricci shrinker satisfies the classical Weyl law [27]. The difficulty is that, although the criterion in [8] is general, it still cannot be applied directly to Ricci shrinkers. The key observation is that, roughly speaking, the problematic locus where this criterion may fail is asymptotically negligible. To establish this, I prove a large-scale regularity estimate showing that, in geodesic balls of radius R, the region with curvature radius below R^{-1} occupies a vanishing proportion of the volume as $R \to \infty$.

My next goal is to go beyond the leading Weyl asymptotics and obtain quantitative remainder estimates. On compact manifolds, refined estimates are derived from the microlocal structure of the wave kernel and Fourier Tauberian arguments, and are closely related to the dynamics of the geodesic flow. Extending these techniques to Schrödinger operators on noncompact manifolds would require controlling simultaneously the Hamiltonian dynamics and the geometry at infinity.

5 Toward a higher-genus Calabi-Yau/Landau-Ginzburg correspondence on the B-side

My long-term goal is to establish a higher-genus Calabi-Yau/Landau-Ginzburg (CY/LG) correspondence on the B-model side, relating a compact Calabi-Yau theory to a noncompact LG model. The correspondence predicts that invariants of a compact Calabi-Yau manifold can be recovered from the singularities of a holomorphic function f on a noncompact space X, providing a bridge between global geometry and singularity theory. It complements mirror symmetry, which relates the complex geometry studied by the B-model to the counting of holomorphic curves on a mirror space. Together, these correspondences offer a route to studying curve-counting problems through singularity theory. The theory is organized by genus. At genus zero, the B-model is governed by tt^*-geometry, which can be viewed as a generalized variation of Hodge structures. With Xinxing Tang, we proved the CY/LG correspondence for Weil-Petersson metrics and tt^*-structures [21].

Genus one. In recent joint work with Xinxing Tang [22], we established the genus-one B-model Calabi–Yau/Landau–Ginzburg correspondence. On the LG side, we constructed a genus-one B-model invariant intrinsically from the LG model, and we proved that it agrees with the corresponding BCOV invariant of the Calabi–Yau hypersurface up to an additive constant that depends only on the dimension. The proof brings together the holomorphic anomaly formula and a detailed analysis of boundary asymptotics on the compactified moduli space, building on our earlier genus-zero correspondence for Weil–Petersson metrics and tt^*-structures [21].

Higher genus. For $g \geq 2$, the next problem is to combine the graph-integral construction of §2 with the noncompact spectral analysis of §4. The new difficulty on the LG side is that vertices may collide while simultaneously escaping to infinity. I plan to construct a compactification recording both phenomena and establish propagator estimates that remain uniform near both boundary strata. The first goal is a canonical regularization of LG graph integrals; their comparison with the compact CY construction is a subsequent problem. A higher-genus B-model CY/LG correspondence, together with its A-model counterpart, would open a route to proving higher-genus mirror symmetry for compact Calabi–Yau manifolds via LG models. Since on LG's side, mirror symmetry is expected to be more tractable. This would in turn supply new methods for computing higher-genus Gromov–Witten invariants.

Together with the genus-one torsion comparison, this program brings together the main techniques developed in my work: configuration-space compactifications for collision singularities, heat-kernel and spectral estimates for noncompact Schrödinger operators, and localization methods associated with the superpotential.

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